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LINEAR ELECTROMECHANICAL RESPONSES OF FERROELECTRIC CERAMICS WITH INDUCTIVE-RESISTIVE EXTERNAL LOAD

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Abstract—We consider here the solution of the transient coupled problem corresponding to the responses of ferroelectric ceramics in the axial mode experiment for which the direction of wave propagation corresponds to the direction of remanent polarization. The external circuit of the problem consists of an inductor and a resistor connected in series. The coupled equations which result from balance of linear momentum and Kirchhoff's law of circuits are solved using Laplace transform techniques and the inverse transforms are obtained via a numerical method developed in a previous paper. The numerical solutions obtained are in qualitative agreement with available experimental results.

1. INTRODUCTION

In this paper we consider the dynamic electromechanical responses of ferroelectric ceramics. It is known that a poled specimen of such a material when subjected to mechanical loads responds electrically, and when subjected to an external electric field responds mechanically. The problem we have in mind corresponds to the axial mode experiment for which the direction of wave propagation corresponds to the direction of remanent polarization, and the external circuit consists of an inductor and resistor connected in series[1]. Our aim is to obtain numerical solutions for this problem and to ascertain the range of results which are to be expected.

The physical basis of our considerations centers on the notion that the ceramic depoles during the course of the experiment. This depoling process results in changing mechanical, piezoelectric and dielectric properties which must be taken into account. The constitutive models which include this and other effects have been proposed by Chen *et al.*[2] and their physical justifications offered by Chen and Peercy[3].

While experimental results corresponding to the problem have been reported (for example, in [4]) no satisfactory theoretical explanations are advanced. Our motivation is to offer what may be regarded as an attempt in obtaining proper formulation and solutions for the problem.

2. CONSTITUTIVE ASSUMPTIONS AND BASIC EQUATIONS

We consider here the linear electromechanical responses of ferroelectric ceramics. The problem we have in mind corresponds to the axial mode experiment. In this experiment a ceramic disc polarized in the axial direction is subjected to time-dependent mechanical loads on either of its electroded surfaces. The surfaces may be connected via some suitable electrical circuit. The resulting electrical output of the ceramic due to electromechanical interactions and the external electrical circuit may be recorded using an appropriate oscilloscope.

The relevant constitutive relations of the stress T and the electric field E at a material point X and time t germane to our present considerations are

$$T(X,t) = \mu(0)S(X,t) + \int_0^t \frac{\mathrm{d}}{\mathrm{d}t}\mu(t-\tau)S(X,\tau)\,\mathrm{d}\tau + \sigma(0)D(t) + \int_0^t \frac{\mathrm{d}}{\mathrm{d}t}\sigma(t-\tau)D(\tau)\,\mathrm{d}\tau, \tag{2.1}$$

$$E(X,t) = \omega(0)S(X,t) + \int_0^t \frac{\mathrm{d}}{\mathrm{d}t}\omega(t-\tau)S(X,\tau)\,\mathrm{d}\tau + \xi(0)D(t) + \int_0^t \frac{\mathrm{d}}{\mathrm{d}t}\xi(t-\tau)D(\tau)\,\mathrm{d}\tau, \tag{2.2}$$

where S is the strain and D is the departure of the electric displacement from some initial electric displacement D_r which characterizes the state of remanent polarization of the ceramic. The strain S is given by the gradient of the mechanical displacement u. During the passage of a mechanical disturbance or the application of an electric field the number of aligned dipoles may be altered, resulting in changes in the material properties. When the number of aligned dipoles decreases or increases the values of the functions μ, σ and ω also decrease or increase so that they may be either relaxation or creep functions; whereas ξ is always a relaxation function. In addition, the ceramic also exhibits mechanical dissipation which, we suspect, dominates the effects of poling or depoling so that μ may be taken as a relaxation function. Note that in (2.1) and (2.2) the electric displacement D is independent of X. This is a consequence of Gauss' law and our assumption that the ceramic does not contain any free charge. Formulae (2.1) and (2.2) are linear versions of those proposed by Chen *et al.* [2]; the physical bases for their proposition are given by Chen and Peercy [3].

The equation of balance of linear momentum together with the constitutive relation of the stress (2.1) yield the usual equation

$$\mu(0)\frac{\partial^2}{\partial X^2}u(X,t)+\int_0^t \frac{d}{dt}\mu(t-\tau)\frac{\partial^2}{\partial X^2}u(X,\tau)\,d\tau=\rho\frac{\partial^2}{\partial t^2}u(X,t) \tag{2.3}$$

for a material exhibiting mechanical dissipation, where ρ is the mass density. However, its solution need not be straightforward because its boundary conditions may depend on D. The governing equation for D is obtained via Kirchhoff's law. We now presume that the disc is bounded by the pair of points (X_1, X_2). Then, for the case of an inductive-resistive external circuit, we have

$$-\int_{X_1}^{X_2} E(X,t)\,dX = i(t)R+\frac{di(t)}{dt}L, \tag{2.4}$$

where i is the current in the external circuit, R and L are its resistance and inductance. Formula (2.4), together with the relation, $i = A\,dD/dt$, with A being the area of each electrode, and the constitutive relation of the electric field (2.2) yield the governing equation for D, viz.

$$AL\frac{d^2D(t)}{dt^2}+AR\frac{dD(t)}{dt}+\xi(0)\Delta D(t)+\Delta\int_{X_1}^{X_2}\int_0^t \frac{d}{dt}\xi(t-\tau)D(\tau)\,d\tau = f(t), \tag{2.5}$$

where $\Delta = X_2 - X_1$, and

$$f(t) = -\omega(0)\int_{X_1}^{X_2}\frac{\partial}{\partial X}u(X,t)\,dX-\int_{X_2}^{X_1}\int_0^t \frac{d}{dt}\omega(t-\tau)\frac{\partial}{\partial X}u(X,\tau)\,d\tau\,dX. \tag{2.6}$$

Notice that ξ, specifically $\xi(0)\Delta$, plays the role of capacitance so that the model resembles an L–C–R circuit. Formulae (2.3) and (2.5) together with the appropriate boundary initial values, e.g.

$$\frac{\partial}{\partial t}u(X_1,T)=\dot{u}_1(t),\quad u(X_2,t)=0,\quad u(X,0)=0,\quad \frac{\partial}{\partial t}u(X,0)=0,$$

$$D(0)=0,\quad \frac{d}{dt}D(0)=0 \tag{2.7}$$

constitute the coupled proplem for the inductance-resistance experiment.

We must now specify the properties of the functions μ, σ, ω and ξ appearing in the constitutive relations (2.1) and (2.2). For the purposes of this paper, we let

$$\mu(s)=(\mu_0-\mu_\infty)\,e^{-s/\tau_\mu}+\mu_\infty,\quad \mu_0>\mu_\infty>0,$$

$$\sigma(s)=(\sigma_0-\sigma_\infty)\,e^{-s/\tau_\sigma}+\sigma_\infty,$$

$$\omega(s)=(\omega-\omega_\infty)\,e^{-s/\tau_\omega}+\omega_\infty,$$

$$\xi(s)=(\xi_0-\xi_\infty)\,e^{-s/\tau_\xi}+\xi_\infty,\quad \xi_0>\xi_\infty>0.$$

Notice that μ and ξ are relaxation functions with relaxation times τ_μ and τ_ξ, and σ and ω are relaxation or creep functions depending on whether $|\sigma_0|$ and $|\omega_0|$ are greater or less than $|\sigma_\infty|$ and $|\omega_\infty|$ with τ_σ and τ_ω being relaxation or retardation times.

We note that if the disc is impacted by a linear elastic material "0" with constant velocity v, then it can be shown that†

$$\dot{u}_1(t) \approx \frac{\rho_0 V_0}{\rho_0 V_0 + \rho V} v + \frac{1}{\rho_0 V_0 + \rho V^2 V_{\dot{u}}^{-1}} \left\{ \int_0^t \int_0^\lambda \frac{d}{d\lambda} \mu(\lambda - \tau) \frac{\partial^2}{\partial X \partial \tau} u(X_1, \tau)\, d\tau\, d\lambda \right.$$
$$\left. + \int_0^t \sigma(t - \tau) \frac{d}{d\tau} D(\tau)\, d\tau \right\}, \qquad (2.8)$$

where $\rho_0 V_0$ is the acoustic impedance of material "O," $\rho V^2 = \mu_0$, and $V_{\dot{u}}$ is the speed of the wave associated with each constant particle velocity. Actually, $V_{\dot{u}}$ depends on the values of μ; but, for convenience, we have taken $V_{\dot{u}}$ to be constant, and we may let $\rho V_{\dot{u}}^2 = \mu_0$ or μ_∞.

3. SOLUTIONS OF EXAMPLE PROBLEMS

It seems that the linearity and convolution integrals of the formulae of Section 3 make the problem represented by (2.7) and (2.8) a neutral candidate for using Laplace transform techniques. However, the transforms are complicated and one does not have much hope of isolating singularities analytically for inversion integral solutions. A numerical method for inverting the transform solutions has been developed in a previous paper[6], and we again appeal to the method in the solution of the present problem.

In order to effect numerical solutions of the problem, we consider a hypothetical material for which‡

$$\rho = 7.8\,\mathrm{gm/cm^3},$$
$$\mu_0 = 158\,\mathrm{GPa}, \quad \mu_\infty = a\mu_0,$$
$$\sigma_0 = \omega_0 = 2.25 \times 10^5\,\mathrm{V/cm}, \quad \sigma_\infty = b\sigma_0, \quad \omega_\infty = c\omega_0,$$
$$\xi_0 = 10^2\,\mathrm{Vcm}/\mu C, \quad \xi_\infty = d\xi_0,$$

where a, b, c and d are assignable constants, and the dimensions of the specimen are $A = 1\,\mathrm{cm}^2$ and $\Delta = 0.25\,\mathrm{cm}$. We also presume that the specimen is impacted by a linear elastic material with velocity $v = 0.01\,\mathrm{cm}/\mu\mathrm{sec}$ and whose acoustic impedance is that of the instantaneous acoustic impedance of the specimen, and we let $\rho V_{\dot{u}}^2 = \mu_0$.

First, we note that for the case of linear piezoelectricity, viz. $a = b = c = d = 1$, and for the situation when $R = 0$ and $L = 0.03\,\mu\mathrm{H}$, our solutions indicate that both the current and the voltage are oscillatory with constant amplitudes, and they are approx. 90° out of phase. Further, since $R = 0$, the voltage oscillates about zero.

We next consider the situation when

$$a = 0.9, \quad \tau_\mu = 0.5\,\mu\mathrm{sec},$$
$$b = c = 0.6, \quad \tau_\sigma = \tau_\omega = 0.01\,\mu\mathrm{sec},$$
$$d = 0.75, \quad \tau_\xi = 0.4\,\mu\mathrm{sec},$$

and $R = 0.5\,\Omega$, $L = 0.03\,\mu\mathrm{H}$. This corresponds to the case of very rapid depoling (manifested by the rather small values for τ_σ and τ_ω). Figures 1 and 2 show that the oscillations of both the current and the voltage are now damped. This damping is caused by the presence of resistance in the external circuit and by the presence of dielectric relaxation. Notice also that the general trends of the current and the voltage are monotonically increasing. This is a consequence of very rapid depoling.

†The derivation of (2.8) follows from the conditions of continuity of stress and mechanical displacement at X_1 and the assumption that the wave propagation in the disc in the vicinity of X_1 is relatively undistorted in the sense of Whitham[5, Chap. 3].

‡These properties are indicative of the responses of a PZT ceramic, a solid solution of lead zirconate and lead titanate.

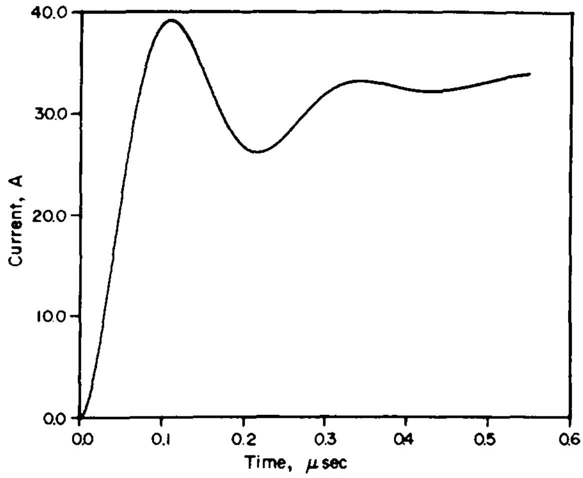

Fig. 1. Current for the case of rapid depoling.

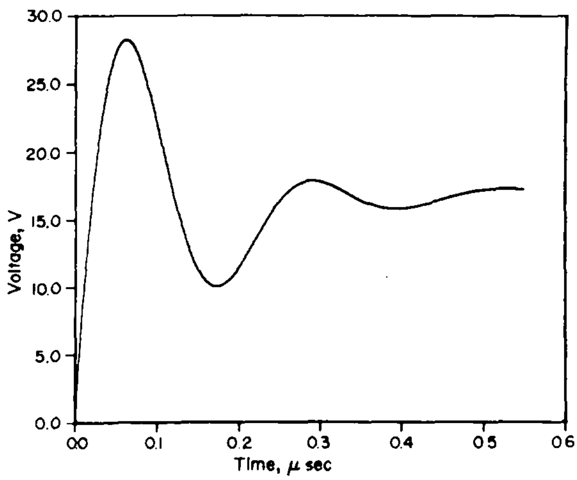

Fig. 2. Voltage for the dase of rapid depoling.

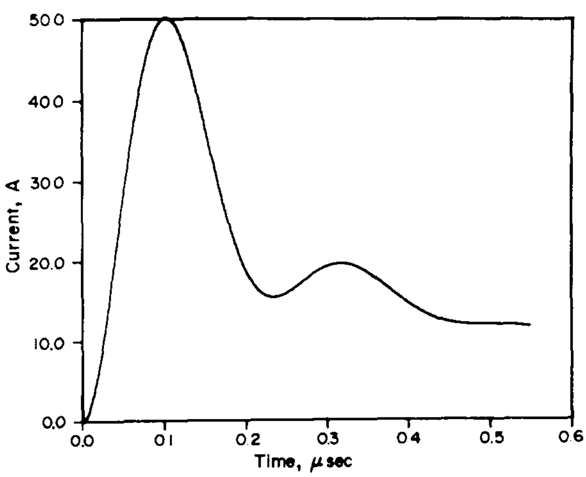

Fig. 3. Current for the case of gradual depoling.

The case of a large amount of depoling and relatively long depoling times is also of interest. In this situation, we let

$$b = c = 0.1;\ \tau_\sigma = \tau_\omega = 0.2\ \mu\text{sec},$$

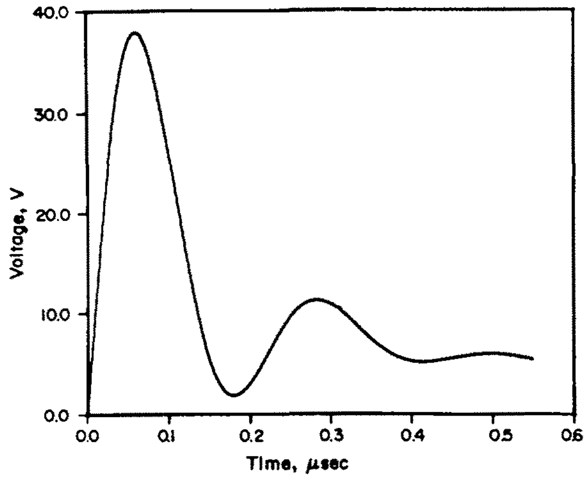

Fig. 4. Voltage for the case of gradual depoling.

and retain the values of all other parameters. Figures 3 and 4 indicate that the oscillations of the current and the voltage are again damped, but now their general trends decrease monotonically after attaining initial maximum values. This, of course, is in contrast to the previous situation.

In closing, we should remark that while we have exhibited the general features of the solutions of the problem, it is clear that a myriad of results are possible depending on the choices of the various parameters. The results shown graphically in the figures, however, are indicative of the type solutions that are expected for this problem. It should also be pointed out that experimental results qualitatively similar to our numerical results have been reported by Lysne[4] but quantitative comparisons are not possible at this time because of the lack of sufficient material information.

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